PRAYAS

JEE 2026

Mathematics

Basic Maths

Lecture -09

By – Ashish Agarwal Sir
(IIT Kanpur)

Topics *to be covered*

- **A** Logarithm and its Properties
- **B** Problem Practice

Homework Discussion

QUESTION

TAH 05

Solve : $(x^2 + 3x + 1)(x^2 + 3x - 3) \geq 5$

$(t+1)(t-3) \geqslant 5$

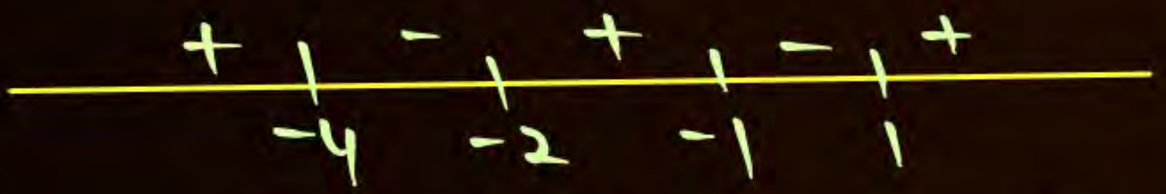
$t^2 - 2t - 3 \geqslant 5$

$t^2 - 2t - 8 \geqslant 0.$

$(t-4)(t+2) \geqslant 0$

$(x^2+3x-4)(x^2+3x+2) \geqslant 0$

$(x+4)(x-1)(x+1)(x+2) \geqslant 0$



$x \in (-\infty, -4] \cup [-2, -1] \cup [1, \infty)$

QUESTION

TAH 06

Find Exhaustive set of values of x satisfying :

(i) $x^3 - 3x^2 - x + 3 > 0$

(ii) $x^4 - 3x^3 - x + 3 < 0$

$x^3(x-3)-1(x-3)<0$

$(x^3-1)(x-3)<0$

$(x-1)\underbrace{(x^2+x+1)}_{D<0,\, a>0 \rightarrow \text{always +ve}}(x-3)<0 \rightarrow (x-1)(x-3)<0$

$x\in(1,3)$

(iii) $x^4 + 6x^3 + 6x^2 + 6x + 5 \le 0$

$x^3(x+1)+5x^2(x+1)+x(x+1)+5(x+1)\le 0$

$(x^3+5x^2+x+5)(x+1)\le 0$

$(x^2(x+5)+1(x+5))(x+1)\le 0$

$\underbrace{(x^2+1)}_{\text{always+ve}}(x+5)(x+1)\le 0$

$(x+5)(x+1)\le 0$ $x\in[5,-1]$

QUESTION

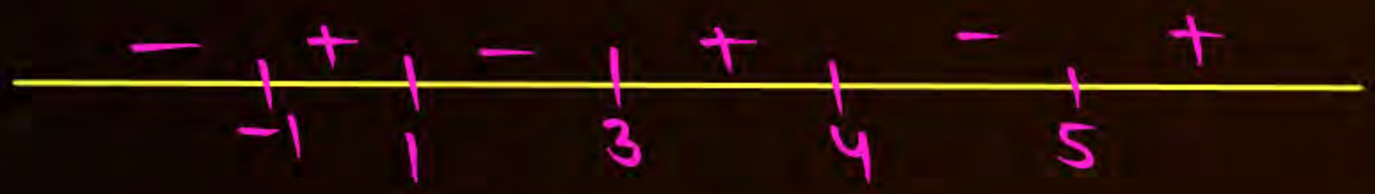
TAH 10

$\left.\begin{matrix}D<0\\ a>0\end{matrix}\right\}$ always +ve

Solve: $\dfrac{(x^2-4x+5)^2(x-3)^2(x+1)^3}{(x-1)(x-5)^3(x^2-7x+12)}>0$

$$\frac{(x-3)^{\cancel{2}}(x+1)^3}{(x-1)(x-5)^3(x-4)\cancel{(x-3)}}>0 \qquad x\neq 3$$

$$\frac{(x-3)(x+1)^3}{(x-1)(x-4)(x-5)^3}>0 \quad , x\neq 3$$



$x\in(-1,1)\cup(3,4)\cup(5,\infty)$

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

QUESTION

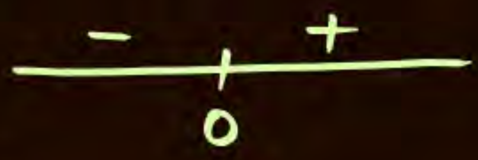
Solve: $x(2^x - 1)(3^x - 9)(x - 3) < 0.$

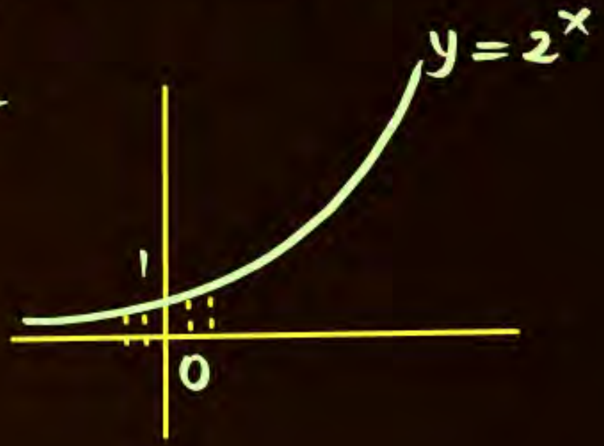
$x \cdot (x-0)\ (x-2)\ (x-3) < 0$

$x = 0$ is N.P

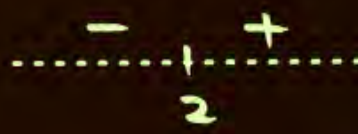
$x \in (2,3)$

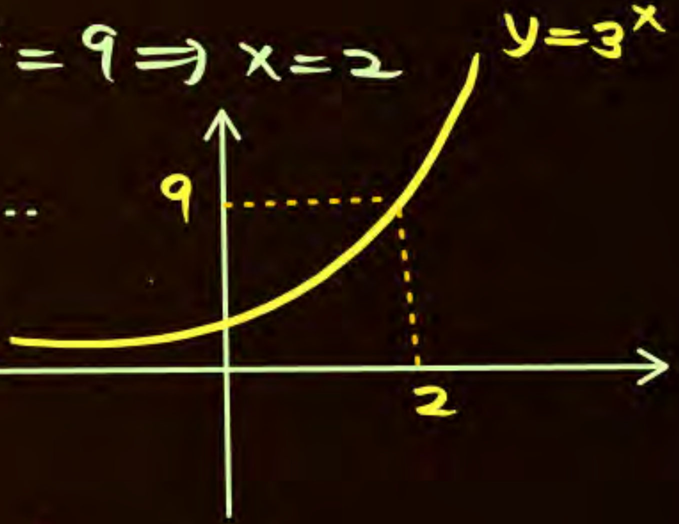
$2^x - 1 = 0 \Rightarrow 2^x = 1 \Rightarrow x = 0$





$3^x - 9 = 0 \Rightarrow 3^x = 9 \Rightarrow x = 2$





Ans. $x \in (2, 3)$

QUESTION

D<0, a>0 always +ve

If $\dfrac{(e^x-1)(2x-3)(x^2+x+2)}{(\sin x-2)(x+1)x} \le 0$ then $x \in$

−ve

A $(-\infty,-1)$

B $\left[\frac{3}{2},\infty\right)$

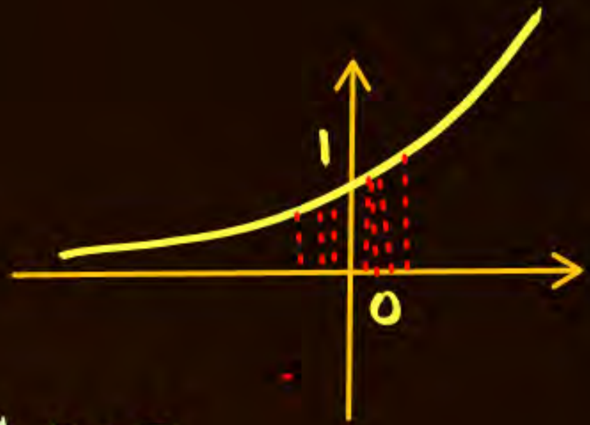
C $(-1,0)$

D $\left(0,\frac{3}{2}\right)$

$\because -1 \le \sin x \le 1$

$-3 \le \sin x - 2 \le -1$

$\sin x - 2 \in R^- \ \forall\ x \in R$



$\dfrac{(e^x-1)(2x-3)}{x(x+1)} \ge 0 \quad \dfrac{\cancel{x}(2x-3)}{\cancel{x}(x+1)} \ge 0$

$e^x-1=0 \Rightarrow x=0$

− +

0

e^x-1 behaves like $(x-0)$

$\dfrac{2x-3}{x+1} \ge 0 \ , \ x \ne 0$

+ − +

−1 3/2

$x \in (-\infty,-1) \cup [3/2,\infty)$

Ans. A, B

QUESTION

Solve: $(x^2 - x - 1)(x^2 - x - 7) < -5$

Tahol

Ans. $x \in (-2, -1) \cup (2, 3)$

QUESTION

Tahoz

If $\frac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0$, then $x \in$

A $(-\infty, -4]$

B $(-1, 0]$

C $(3, \infty)$

D $\{1\}$

Ans. A, B, C, D

QUESTION

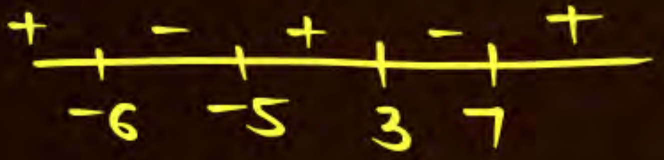
TAH 03

1. Solve $\frac{x(3-4x)(x+1)}{(2x-5)} < 0$ [Ans. $x \in (-\infty, -1) \cup (0, 3/4) \cup (5/2, \infty)$]

2. Solve $\frac{(2x+3)(4-3x)^3(x-4)}{(x-2)^2 x^5} \leq 0$ [Ans. $x \in (-\infty, -3/2) \cup (0, 4/3] \cup [4, \infty)$]

3. Solve $\frac{(x-3)(x+5)(x-7)}{|x-4|(x+6)} \leq 0$ [Ans. $x \in (-6, -5] \cup [3, 4) \cup (4, 7]$]

$$\xrightarrow{} \frac{(x-3)(x+5)(x-7)}{(x+6)} \leq 0, \quad x \neq 4$$



$$x \in (-6, -5] \cup [3, 7] - \{4\}$$

OR

$$x \in (-6, -5] \cup [3, 4) \cup (4, 7]$$

4. Solve $\frac{5x+1}{(x+1)^2} < 1$ [Ans. $x < 0$ or $x > 3, x \neq -1$]

5. Solve $\frac{x^4}{(x-2)^2} > 0$ [Ans. $x \in R - \{0, 2\}$]

6. Solve $\frac{6x^2-5x-3}{x^2-2x+6} \leq 4$ [Ans. $-\frac{9}{2} \leq x \leq 3$]

7. Solve $\frac{(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$ [Ans. $x \in (-\infty, -2] \cup \{1\}$]

Logarithm

was invented to make calculations easier

logarithm converts product in to sum.

& converts division in to difference

Which would you Prefer to Attempt

(a) (37.53×8.74) or $37.53 + 8.74$

(b) $56.98 \div 12.76$ or $56.98 - 12.73$

Definition

Input of logarithm

if $a^x = N \iff \log_a N = x$ value of logarithm

$(a>0,\ a\neq 1,\ N>0,\ x\in R)$ Base of logarithm

Ex: $\log_2 8 = 3$

Ex: $\log_2 x = -1$

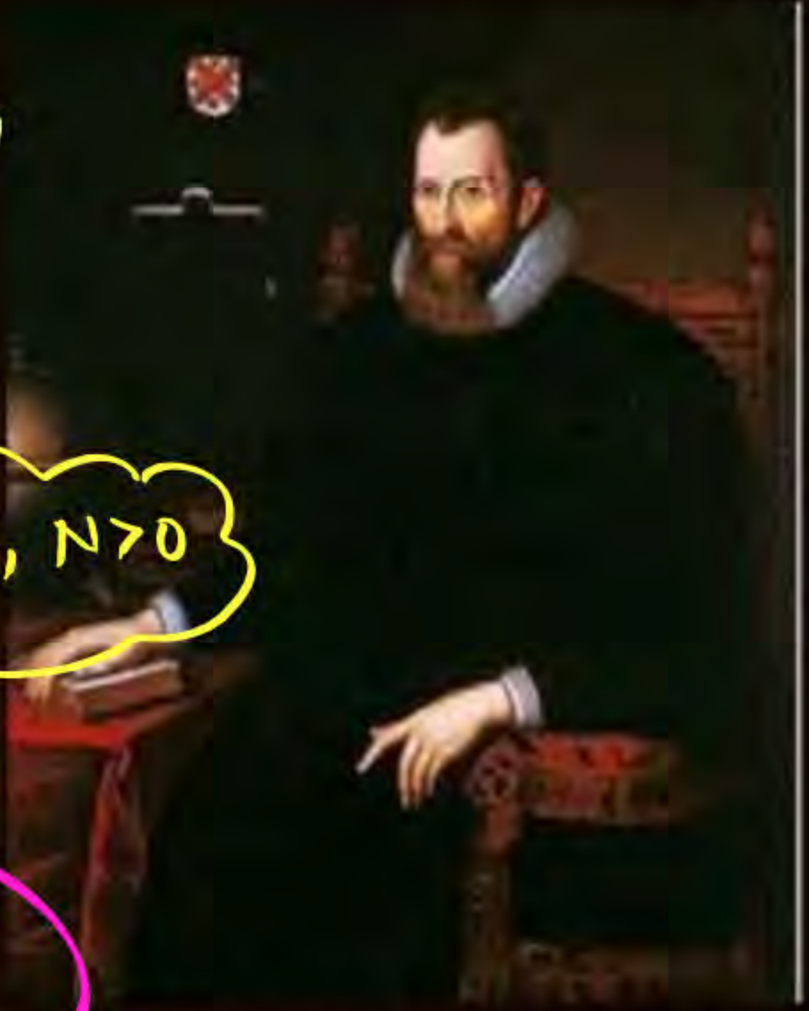
find x : $2^{-1} = x$

$x = \frac{1}{2}$

$\log_a N$ is defined only if $a>0,\ a\neq 1,\ N>0$

★ $\log_a N$ kaa Value voh power hoti hai jisay 'a' pe lagayaa jayay Taaki 'N' aa jayay

★ $\log_a N = x \iff a^x = N$



JOHN NAPIER

Evaluate: $\log_{32} 128$

let $x = \log_{32} 128$

$$128 = 32^x$$

$$2^7 = 2^{5x}$$

$$5x = 7$$

$$\boxed{x = 7/5}$$

- $\log_a a = 1$
- $\log_a 1 = 0$
- $\log_a (1/a) = -1$

- $\log_{\frac{1}{a}} a = -1$

Evaluate: $\log_{10}(.01)$

$$\log_{10}\frac{1}{100} = \log_{10} 10^{-2} = -2$$

let $\log_{10}(0.01) = x$

$$0.01 = 10^x$$

$$\frac{1}{100} = 10^x$$

$$10^x = 10^{-2}$$

$$\boxed{x = -2}$$

✱ $a^{\left(\log_a x\right)} = x$

proof:

let $\log_a x = t \Rightarrow a^t = x$

Now taking LHS

$a^{\log_a x} = a^t = x$ = RHS (proved)

Ex: $\textcircled{2}^{\log_{\textcircled{2}} 100} = 100$

Nichod!!

(a) $\log_N N = 1$ i.e. logarithm of a number to the same base is 1.

(b) $\log_{\frac{1}{N}} N = -1$ i.e. logarithm of a number of its reciprocal –1.

(c) $\log_a 1 = 0$ i.e. logarithm of unity to any base is zero.

(d) $a^{\log_a N} = N$ is a identify for all $N > 0$ and $a > 0, a \neq 1$

e.g. $2^{\log_2 5} = 5$

QUESTION

Column 1	Column 2
(a) $\log_{16}32$ Ⓢ	(P) -1
(b) $\log_{9}27$ Ⓣ	(Q) 1
(c) $\log_{2}(\log_{2}4) = \log_2 2 = 1$ Ⓠ	(R) 2
(d) $\log_{2-\sqrt{3}}(2+\sqrt{3}) = -1$ Ⓟ	(S) $5/4$
(e) $\log_{5\sqrt{5}}125 = 2$ Ⓡ	(T) $3/2$

ⓐ $\log_{16}32 = x$

$32 = 16^x$

$2^5 = 2^{4x}$

$x = 5/4 \rightarrow$ Ⓢ

ⓑ $\log_{9}27 = x$

$9^x = 27$

$3^{2x} = 3^3$

$x = 3/2 \rightarrow$ Ⓣ

$(2+\sqrt{3})(2-\sqrt{3}) = 1$

$2+\sqrt{3}$ & $2-\sqrt{3}$ are reciprocal of Each other.

$\log_2 4 = 2$, $\log_3 27 = 3$, $\log_{\frac{1}{2}} 1/8 = 3$, $\log_{\frac{1}{3}} \frac{1}{9} = 2$

$a, N > 1$

$0 < a, N < 1$

$\log_2 \frac{1}{4} = -2$, $\log_{\frac{1}{3}} 27 = -3$, $\log_{1/2} 8 = -3$

$a > 1, 0 < N < 1$

$0 < a < 1, N > 1$

$\log_a N = +ve$

$\log_a N = -ve$

$\log_a x = \log_a y \Rightarrow x = y$

$\log_a x = \log_a y = \lambda$

$x = a^\lambda = y$

★ If $x = y \Longrightarrow \log_a x = \log_a y$ (False)

★ $\underline{\text{If } x, y \in R^+}$ & $x = y$ then $\log_a x = \log_a y$ $(a > 0, a \neq 1)$ (True)

NOTE:

1. It must be noted that whenever the number and the base are on the same side of unity the value of logarithm is positive, however if the number and the base are located on different side of unity then the value of logarithm is negative.

2. If two number are equal then their logarithm to the same base are equal and conversely.

QUESTION

Find the value of x.

(1) $\log_{(5-x)}(x^2 - 2x + 65) = 2$

$$(5-x)^2 = x^2 - 2x + 65$$

$$25 + \cancel{x^2} - 10x = \cancel{x^2} - 2x + 65.$$

$$8x = -40$$

$$x = -5 ✓$$

(2) $\log_{(x-1)}(4) = 2$

$$(x-1)^2 = 4$$

$$x - 1 = -2, 2$$

$$x = \cancel{-1}, 3 ✓$$

Base becomes -ve.

QUESTION

Find the value of x.

(3) $\log_3(3^x - 6) = x - 1$ $\longrightarrow$ $3^x - 6 = 3^{x-1}$

(4) $\log_2(4 + \log_3 x) = 3$

$3^x - 6 = 3^x \cdot 3^{-1}$

$3^x - 6 = \frac{3^x}{3}$

let $3^x = t$

$t - 6 = \frac{t}{3}$

$3t - 18 = t$

$t = 9$

$3^x = 9$

$x = 2.$

$4 + \log_3 x = 2^3$

$\log_3 x = 8 - 4 = 4$

$x = 3^4$

$x = 81$

QUESTION

Find the value of x.

(5) $\log_2(x+1) - \log_2(2x-3) = 0$

$$\log_2(x+1) = \log_2(2x-3)$$

$$x+1 = 2x-3$$

$$\boxed{x=4}$$ ✓

QUESTION

Find all values of x for which the following equalities hold true.

(i) $\log_2 x^2 = 1$

(ii) $\log_3 x = \log_3 (2 - x)$

(iii) $\log_4 x^2 = \log_4 x$

(iv) $\log_{1/2}(2x + 1) = \log_{1/2}(x + 1)$

(v) $\log_{1/3}(x^2 + 8) = -2$

Tah 04

QUESTION

Solve: $7^{\log_7 x} + 2x + 9 = 0$

$x + 2x + 9 = 0$

$x = -3$ Ans → Gadho / Gadhiyoo aisaa naa karo

correct: $x \in \phi$

$\because$ at $x = -3$ $\log_7 x$ is not defined.

The Principal Properties of Logarithm

If m, n are arbitrary positive numbers where, $a > 0$, $a \neq 1$ and x is any real numbers,

(1) $\log_a mn = \log_a m + \log_a n$

(2) $\log_a \frac{m}{n} = \log_a m - \log_a n$

(3) $\log_a m^x = x\log_a m$

proof: let $\log_a m = x$, $\log_a n = y$

$a^x = m$, $a^y = n$

$a^{x+y} = m \cdot n$, $\frac{a^x}{a^y} = \frac{m}{n}$

$\log_a mn = x+y$, $a^{x-y} = m/n$

$\log_a mn = \log_a m + \log_a n$.

$\log_a (m/n) = x-y$

$\log_a m/n = \log_a m - \log_a n$

proof: let $\log_a m = \alpha$

$a^\alpha = m$.

$a^{\alpha x} = m^x$

$\log_a m^x = \alpha x$

$\log_a m^x = x\alpha = x\log_a m$

QUESTION

Find the value of following

(1) $\log_{39}\frac{15}{7}+\log_{39}\frac{13}{3}-\log_{39}\frac{5}{21}$ — $\log_{39}\left(\frac{15}{7}\cdot\frac{13}{3}\div\frac{5}{21}\right)=\log_{39}\left(\frac{15}{7}\times\frac{13}{3}\cdot\frac{21}{5}\right)=\log_{39}39=1$

(2) $2\log_6 2+3\log_6 3+\log_6 12$

$\|$

$\log_6 2^2+\log_6 3^3+\log_6 12$

$\|$

$\log_6(2^2\cdot 3^3\cdot 12)=\log_6(2^2\cdot 3^3\cdot 2^2\cdot 3^1)$

$=\log_6(2^4\cdot 3^4)$

$=\log_6(2\times 3)^4=4\log_6 6=4$

QUESTION

Find: $\log_2[\log_2\{\log_3(\log_3 27^3)\}]$

$\log_2(\log_2(\log_3(\log_3 3^9)))$

$\log_2(\log_2(\log_3 9))$

$\log_2(\log_2 2)$

$\log_2 1 = 0.$

QUESTION

Solve:

(i) $2^{\log_2 x^2} - 3x - 4 = 0;$

(ii) $2^{2\log_2 x} - 3x - 4 = 0$

$x^2 - 3x - 4 = 0 \quad (\because a^{\log_a x} = x)$

$(x-4)(x+1) = 0$

$x = 4, -1$

$2^{\log_2 x^2} - 3x - 4 = 0$

$x^2 - 3x - 4 = 0$

$x = 4, \cancel{-1}$

$\log_2 16 = \log_2 2^4 = 4\log_2 2 = 4$

$\shortparallel$

$\log_2 (-2)^4 \neq 4\log_2(-2)$

$\shortparallel$

$4\log_2|-2| = 4\cdot\log_2 2 = 4$

$$\log_a x^{2n} = 2n \cdot \log_a |x|$$

ⓐ $\log_a mn$ can be written as. $\log_a m + \log_a n$ (False)

ⓑ $\log_a m + \log_a n$ can be written as: $\log_a mn$ (True)

$$\log_2((-2)\cdot(-4)) \neq \log_2(-2) + \log_2(-4)$$

find Domain of $f(x)=\log_{10}(x-1)+\log_{10}(x-4)$

$x-1>0$ & $x-4>0$

$x>4$

$D_f : (4,\infty)$

find Domain of $g(x)=\log_{10}((x-1)\cdot(x-4))$

$\log_a N$ is defined only if $a>0,\ a\neq 1$ & $N>0$

Domain : $x\in(-\infty,1)\cup(4,\infty)$

QUESTION

If a, b and c are positive real numbers such that

$a^{\log_3 7} = 27; b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$.

Find the value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$.

$$E = \left(a^{\log_3 7}\right)^{\log_3 7} + \left(b^{\log_7 11}\right)^{\log_7 11} + \left(c^{\log_{11} 25}\right)^{\log_{11} 25}$$

$$= \boxed{27}^{\log_3 \boxed{7}} + \boxed{49}^{\log_7 \boxed{11}} + \sqrt{11}^{\log_{11} 25}$$

$$\because \ a^{\log_b c} = c^{\log_b a}$$

$$= 7^{\log_3 27} + 11^{\log_7 49} + 25^{\log_{11} \sqrt{11}}$$

$$= 7^3 + 11^2 + 25^{1/2} = 469.$$

Chamatkaari BABA naa Banay.......

- $\log_a (m + n) \neq \log_a m + \log_a n$
- $\log_a (m + n) \neq \log_a m \cdot \log_a n$
- $\log_3 (9x) \neq 2 \log_3 3x$
- $\log_a^n x \neq n \log_a x$

$$\log_a^2 x = (\log_a x)^2$$

$$\log_a^3 x = (\log_a x)^3$$

$$\log_a^n x = (\log_a x)^n$$

Base Changing Theorem

$$\frac{\log_c a}{\log_c b} = \log_b a \rightarrow \text{proof:}$$

$$\boxed{\log_b a = \frac{\log_c a}{\log_c b}}$$

let $\log_b a = \alpha$

$a = b^\alpha$

LHS

$$\frac{\log_c a}{\log_c b} = \frac{\log_c b^\alpha}{\log_c b} = \frac{\alpha \cancel{\log_c b}}{\cancel{\log_c b}} = \alpha = \log_b a = \underline{\text{RHS}}$$

Ex: $\log_{32} 64$

$$\| $$

$$\frac{\log_2 64}{\log_2 32}$$

$$\|$$

$$\frac{\log_2 2^6}{\log_2 2^5} = \frac{6 \log_2 2}{5 \log_2 2}$$

$$= \underline{6/5.}$$

Important deduction from base changing theorem

$(a, b > 0,\ a, b \neq 1)$

D₁ : $\log_b a = \dfrac{1}{\log_a b}$ — proof: $\log_b a = \dfrac{\log_a a}{\log_a b} = \dfrac{1}{\log_a b}$

Interchangeable

$a^{\log_b c} = c^{\log_b a}$ — $(a, b, c > 0,\ b \neq 1)$

D₂ : $a^{\log_b c} = c^{\log_b a}$

proof:

$$a^{\log_b c} = a^{\frac{\log_a c}{\log_a b}} = \left(a^{\log_a c}\right)^{\frac{1}{\log_a b}} = c^{\log_b a}$$

Saari Class Illustrations
Retry karni Hai

QUESTION

(Home Challenge-02)

Solve in real numbers the equation

$\sqrt{x_1 - 1} + 2\sqrt{x_2 - 4} + \cdots + n\sqrt{x_n - n^2} = \frac{1}{2}(x_1 + x_2 + \cdots + x_n)$ for $x_1, x_2, x_3 \ldots, x_{n-1}, x_n$.

Solution to Previous TAH

QUESTION

TAH 01

Solve: $(x - 1)(x^2 + 4x + 1)(x + 2) \leq 0$

Q Solve :-

$$(x-1)\,(x^2+4x+1)\,(x+2) \le 0$$

Tah-01

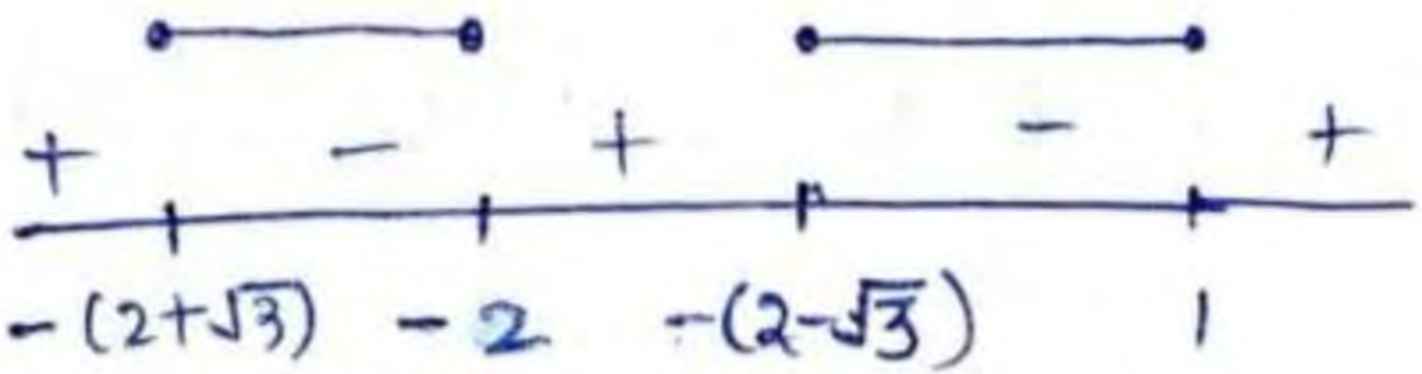
Sol

$$\hookrightarrow D = 16-4 = 12 > 0$$

$$\alpha, \beta = \frac{-4 \pm \sqrt{12}}{2} = -2 \pm \sqrt{3}$$

$$(x-1)\,(x-(-2+\sqrt{3}))\,(x-(-2-\sqrt{3}))\,(x+2) \le 0$$

$$(x-1)\,(x+2-\sqrt{3})\,(x+2+\sqrt{3})\,(x+2) \le 0$$



$$x \in [-(2+\sqrt{3}), -2] \cup [-(2-\sqrt{3}), 1]$$ Ans

Date. ___/___/___

TAH-01

$$(x-1)\,(x^2+4x+1)\,(x+2)\leq 0$$

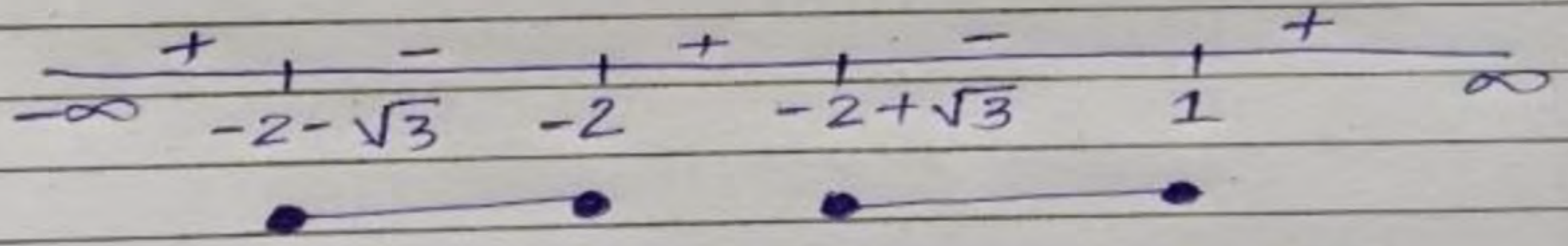
$$\hookrightarrow D = 16-4\times1\times1$$

$$D > 0$$

$$\alpha,\beta = \frac{-4\pm\sqrt{12}}{2}$$

$$\alpha,\beta = -2\pm\sqrt{3}$$

$$\Rightarrow (x-1)\,(x-(-2+\sqrt{3}))\,(x-(-2-\sqrt{3}))\,\underset{-ve}{(x+2)\leq 0}$$



$x\in[-2-\sqrt{3}\,,-2]\cup[-2+\sqrt{3}\,,\,1]$ Ans

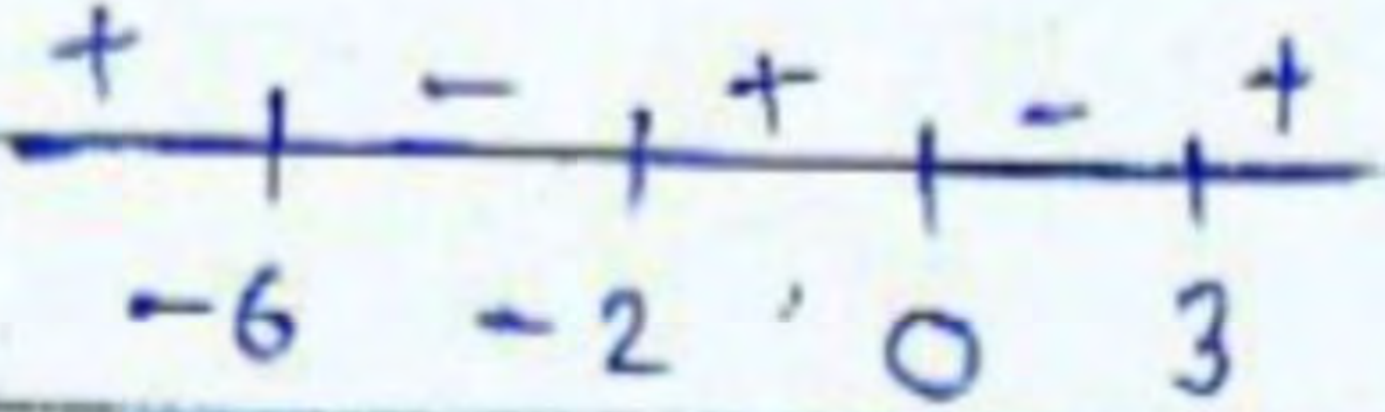
QUESTION

TAH 02

Solve: $(x^2 - x - 6)(x^2 + 6x) \geq 0$

Tah2 → $(x^2-x-6)(x^2+6x) \geq 0$

$(x-3)(x+2)x(x+6) \geq 0$



$(-\infty,-6] \cup [-2,0] \cup [3,\infty)$

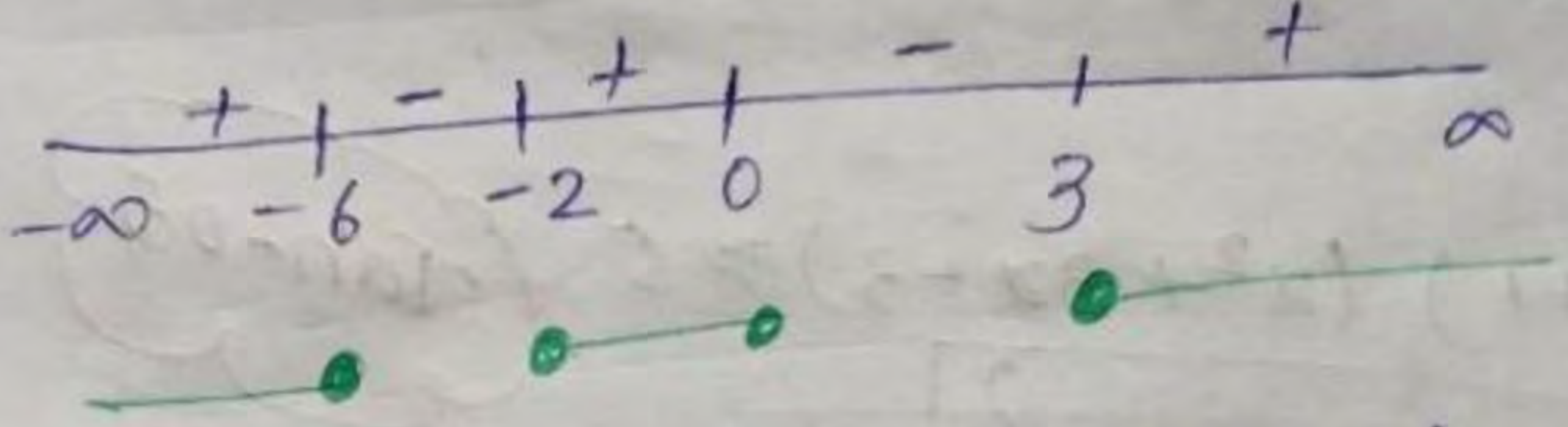
Q. $(x^2-x-6)(x^2+6x) \geq 0$

Tah-02

Sol^n.

$(x^2-3x+2x-6)(x(x+6)) \geq 0$

$(x-3)(x+2)(x)(x+6) \geq 0 \longrightarrow +ve$



$x \in (-\infty, -6] \cup [-2, 0] \cup [3, \infty)$ Ans

QUESTION

TAH 03

Solve: $x^2 - 5x + 6 \geq 0$ and $x^2 - 10x + 24 \leq 0$

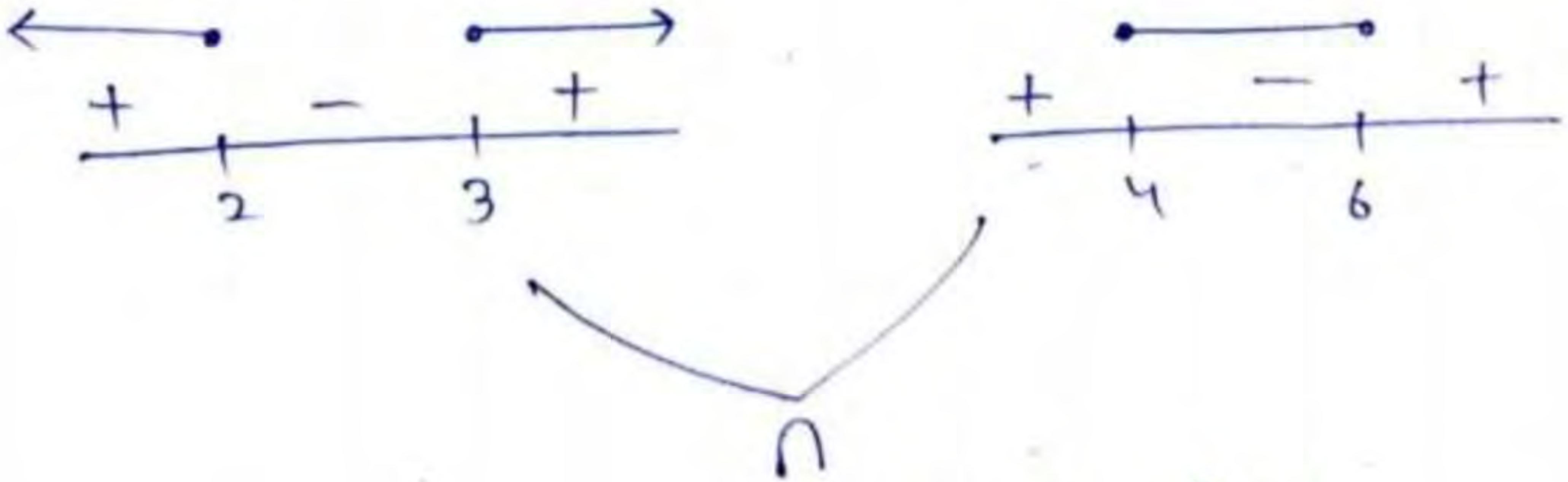
Q Solve: Tah-03

$x^2 - 5x + 6 \geqslant 0$ and $x^2 - 10x + 24 \leq 0$

$x^2 - 3x - 2x + 6 \geqslant 0$ $\qquad$ $x^2 - 6x - 4x + 24 \leq 0$

$x(x-3) - 2(x-3) \geqslant 0$ $\qquad$ $x(x-6) - 4(x-6) \leq 0$

$(x-2)(x-3) \geqslant 0$ $\qquad$ $(x-4)(x-6) \leq 0$



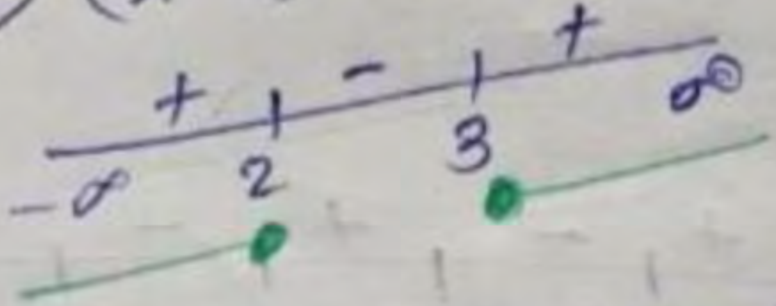
$x \in [4, 6]$ Ans

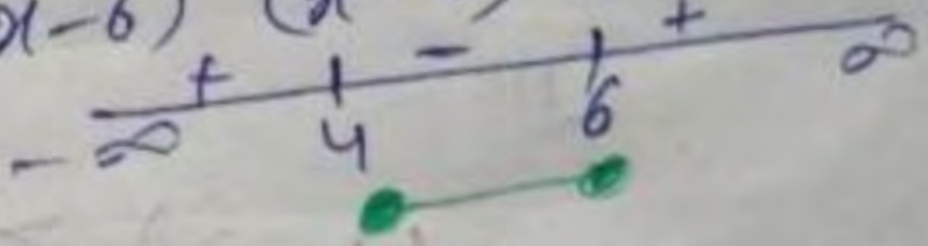
Q $x^2-5x+6 \geq 0$ and $x^2-10x+24 \leq 0$ Tah-03

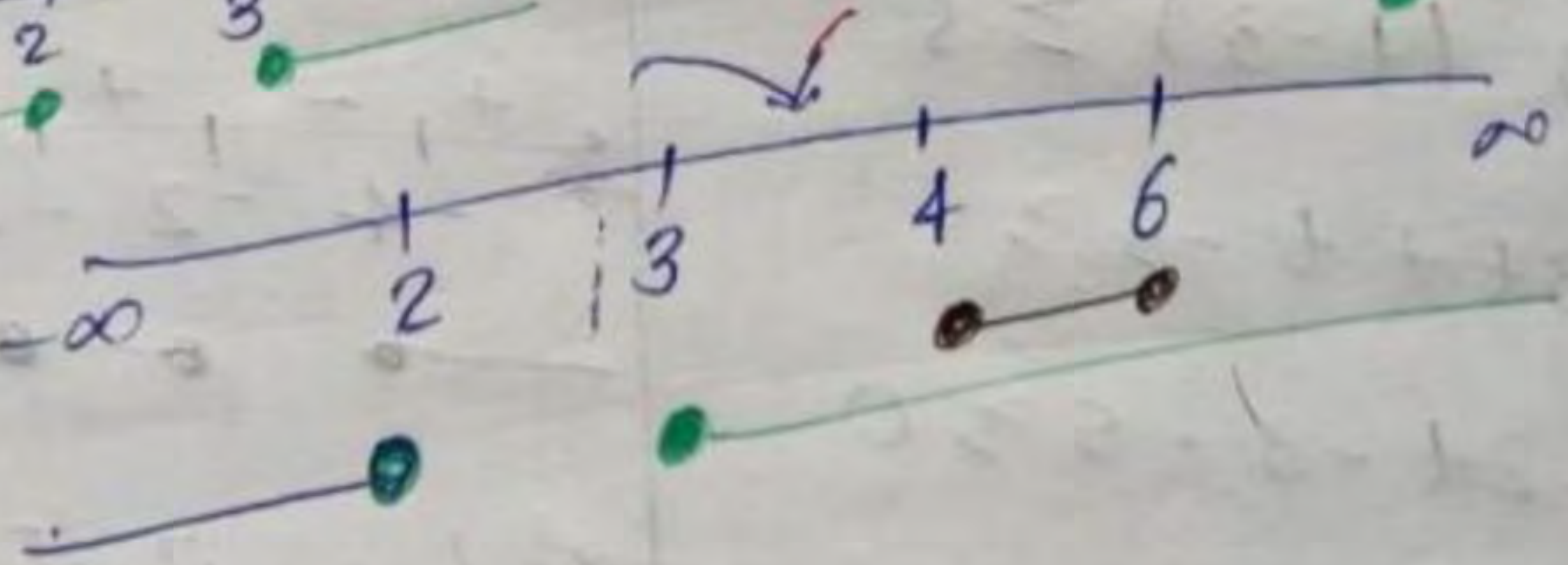
$\Downarrow$

$\Rightarrow x^2-3x-2x+6 \geq 0$ $\quad$ $x^2-6x-4x+24 \leq 0$

$\Rightarrow (x-3)(x-2) \geq 0$ $\quad$ $(x-6)(x-4) \leq 0$







$x \in [4, 6]$ Ans

QUESTION

TAH 04

$$\frac{x^2 - x - 6}{x^2 + 6x} \geq 0$$

18

$$\frac{x^2-x-6}{x^2+6x} \geq 0$$

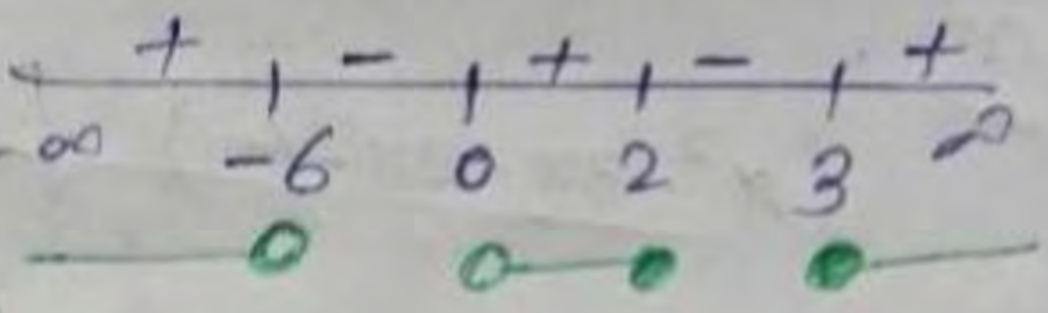
Tah-04

Sol^n

$$\frac{x^2-3x+2x-6}{x(x+6)} \geq 0$$

$$\frac{(x-3)(x-2)}{(x)(x+6)} \geq 0$$

↳ +ve



$x \in (-\infty, 6) \cup (0, 2] \cup [3, \infty)$

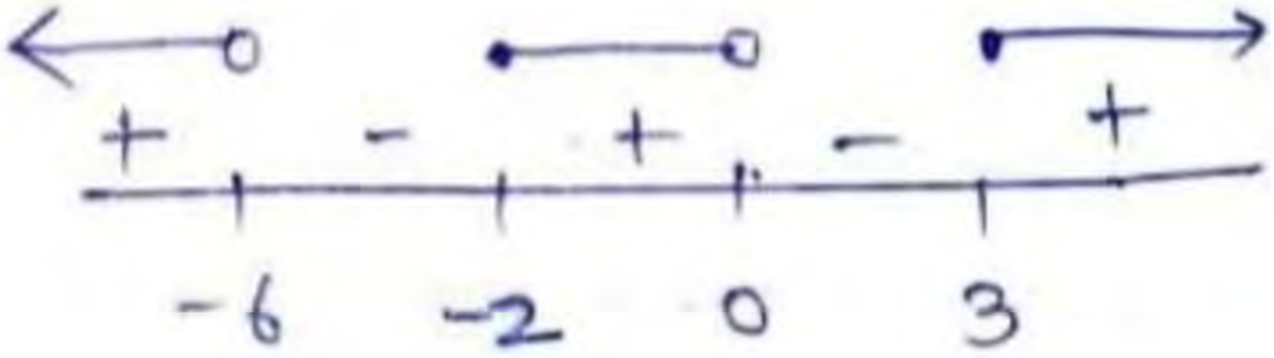
Ans

Q $$\frac{x^2-x-6}{x^2+6x} \geq 0$$

Sol

$$\frac{x^2-3x+2x-6}{x(x+6)} = \frac{(x-3)(x+2)}{x\ (x+6)} \geq 0$$

Tanoy



$x \in (-\infty, 6) \cup [-2, 0) \cup [3, \infty)$ Ans

QUESTION

TAH 05

Solve : $(x^2 + 3x + 1)(x^2 + 3x - 3) \geq 5$

• Q-5: Solve: $(x^2+3x+1)(x^2+3x-3) \geqslant 5$.

Soln: let $\boxed{x^2+3x=t}$.

TAH 5
BY REED
FROM WB

So, $(t+1)(t-3) \geqslant 5$

$\Rightarrow t^2-2t-3-5 \geqslant 0$

$\Rightarrow t^2-2t-8 \geqslant 0$

$\Rightarrow (t-4)(t+2) \geqslant 0$

$\Downarrow$

$\therefore t \in (-\infty,-2] \cup [4,\infty)$

$t \leqslant -2$ or $t \geqslant 4$

or, $x^2+3x \leqslant -2$

$\Rightarrow x^2+3x+2 \leqslant 0$

$\Rightarrow (x+1)(x+2) \leqslant 0$

$\Downarrow$

$x \in [-2,-1]$

or, $x^2+3x \geqslant 4$

or, $x^2+3x-4 \geqslant 0$

or, $(x+4)(x-1) \geqslant 0$.

$\Downarrow$

$x \in (-\infty,-4] \cup [1,\infty)$

Union

$\therefore x \in (-\infty,-4] \cup [-2,-1] \cup [1,\infty)$

Q $(x^2+3x+1)(x^2+3x-3) \geqslant 5$

Tah os

Sol Let, $x^2+3x=t$

$(t+1)\ (t-3) \geqslant 5$

$t^2-3t+t-3 \geqslant 5$

$t^2-2t-8 \geqslant 0$

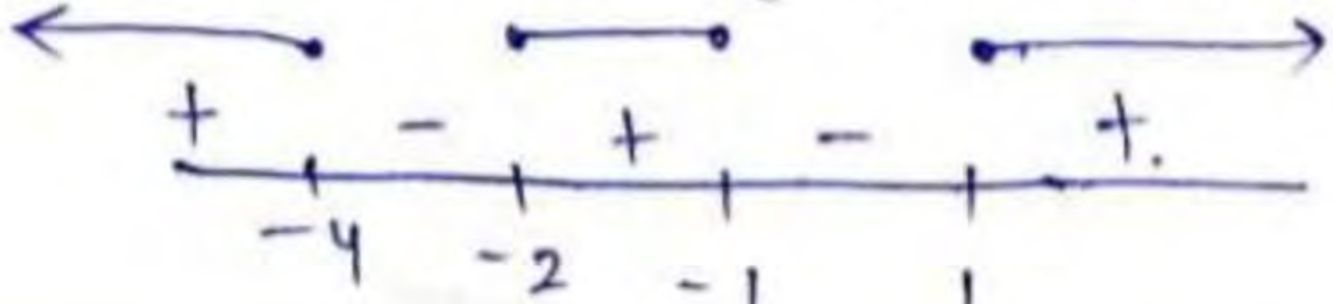
$(t-4)\ (t+2) \geqslant 0$

$(x^2+3x-4)\ (x^2+3x+2) \geqslant 0$

$(x^2+4x-x-4)\ (x^2+2x+x+2) \geqslant 0$

$x(x+4)-1(x+4) \quad x(x+2)+1(x+2) \geqslant 0$

$(x-1)\ (x+4) \quad (x+1)\ (x+2) \geqslant 0$



$x \in (-\infty,-4] \cup [-2,-1]$
$\cup\ [1,\infty)$

QUESTION

TAH 06

Find Exhaustive set of values of x satisfying :

(i) $x^3 - 3x^2 - x + 3 > 0$

(ii) $x^4 - 3x^3 - x + 3 < 0$

(iii) $x^4 + 6x^3 + 6x^2 + 6x + 5 \leq 0$

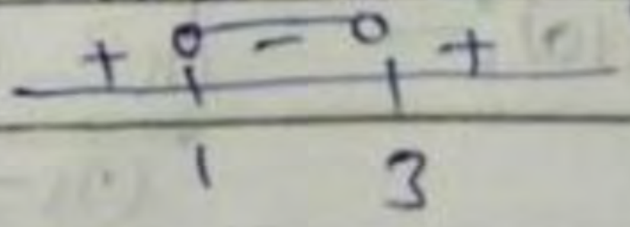
TAH 6 :— (ii) $x^4 - 3x^3 - x + 3 < 0$

$$x^3(x-3) - 1(x-3) < 0$$

$$\Rightarrow (x-3)(x^3-1) < 0$$

$$\Rightarrow (x-3)(x-1)\underbrace{(x^2+x+1)}_{D<0,\ a>1 \text{ always +ve}} < 0$$

$$\Rightarrow (x-3)(x-1) < 0$$



$$x \in (1,3)$$

(iii) $x^4 + 6x^3 + 6x^2 + 6x + 5 \leq 0$

by tit & trial we get $(x+1)$ is factor

$$x^3(x+1) + 5x^2(x+1) + x(x+1) + 5(x+1) \leq 0$$

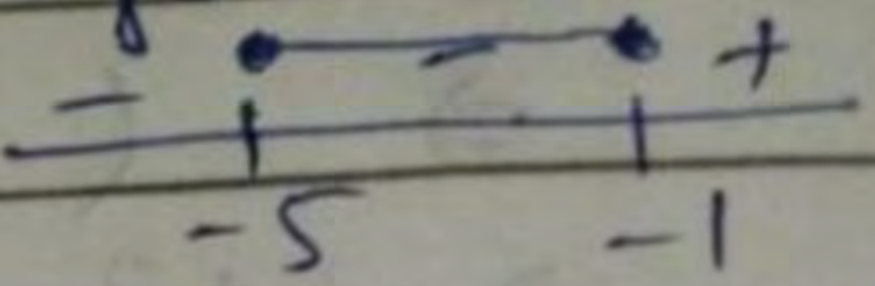
$$(x+1)(x^3+5x^2+x+5) \leq 0$$

$$(x+1)(x^2(x+5)+1(x+5)) \leq 0$$

$$\Rightarrow (x+1)(x+5)\underbrace{(x^2+1)}_{D<0,\ a>0, \text{ always +ve}} \leq 0$$

$$(x+1)(x+5)\underbrace{(x^2+1)}_{\text{always +ve}} \le 0$$

$$(x+1)(x+5) \le 0$$



$x \in [-5,-1]$

Q-6! (i) $x^4-3x^3-x+3<0$

(ii) $x^4+6x^3+6x^2+6x+5\le 0$.

TAH 6-(i)

Soln → (i) $x^4-3x^3-x+3<0$

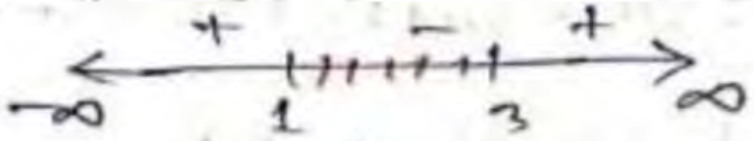
or $x^3(x-3)-1(x-3)<0$

or $(x-3)(x^3-1)<0$

or. $(x-3)(x-1)\underbrace{(x^2+x+1)}_{\hookrightarrow D<0,\ a>0 \Rightarrow \text{always +ve.}}<0$

or, $(x-3)(x-1)<0$.

$\boxed{\therefore x\in(1,3)}$



→(ii) $x^4+6x^3+6x^2+6x+5\le 0$.

$\Rightarrow x^2(x^2+6x+5)+(x^2+6x+5)\le 0$.

$\Rightarrow (x^2+6x+5)\underbrace{(x^2+1)}_{\text{always +ve}}\le 0$.

$\Rightarrow (x+5)(x+1)\le 0$.

$\Downarrow$

$\therefore \boxed{x\in[-5,-1]}$.

TAH 6-(ii)
BY REED
FROM WB

QUESTION

TAH 07

Solve: $(x - 1)^2 (x + 1)^3 (x - 4) < 0$

⑦ $(x-1)^2(x+1)^3(x-4)<0$

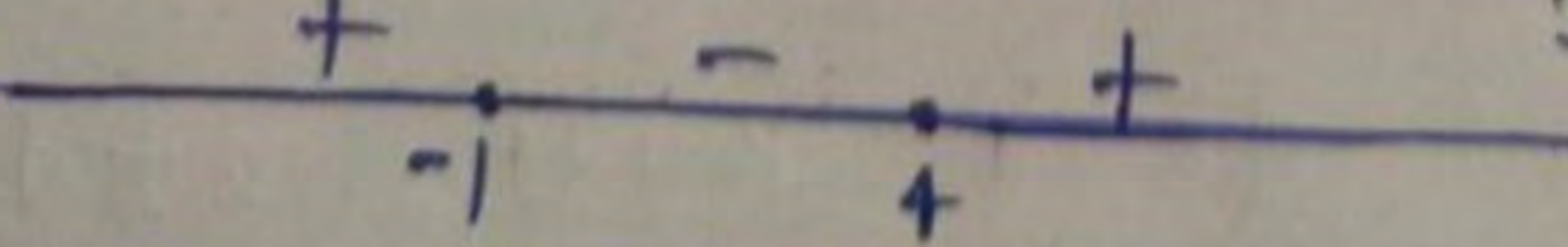
$(x+1)^3(x-4)<0$

$x \neq 1$

$x \in (-1,4)-\{1\}$

Tah-07

$$(x-1)^2 (x+1)^3 (x-4) < 0$$



$x \neq 1$

$$x \in (-1, 4) - \{1\}$$

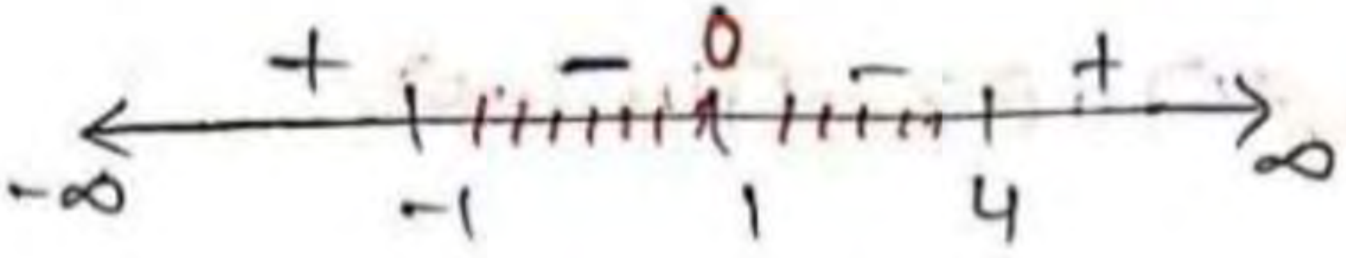
Q-7! Solve: $(x-1)^2(x+1)^3(x-4)<0$

TAH 7

Soln: $(x-1)^2(x+1)^3(x-4)<0$

$$\Downarrow$$

$(x-1)^2(x+1)(x-4)<0$ & $x\neq 1$.



$\therefore x\in(-1,1)\cup(1,4)-\{1\}$

already not present

$$\Downarrow$$

$x\in(-1,1)\cup(1,4)$

QUESTION

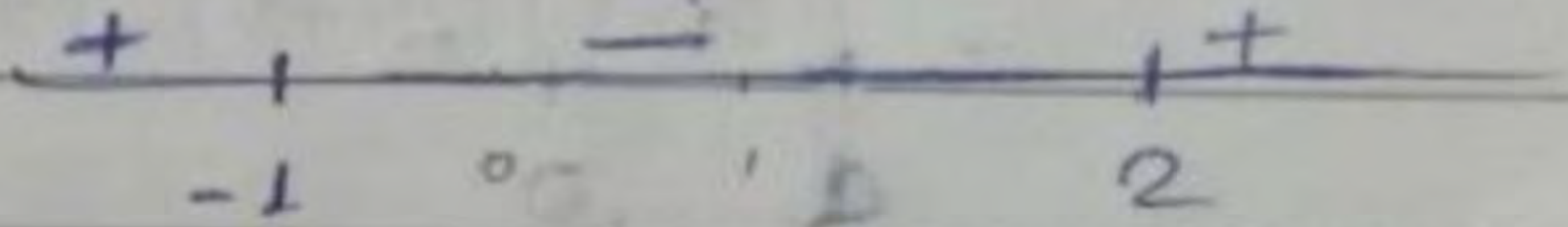
TAH 08

Solve: $\frac{(x-1)^2(x+1)^3}{x^4(x-2)} \leq 0$

⑧ $$\frac{(x-1)^2\,(x+1)^3}{x^4\,(x-2)} \le 0$$

$$\frac{\cancel{(x-1)^2}\,(x+1)^3}{(x-2)} \le 0 \qquad x = 1 \checkmark$$

$$x \ne 0$$



$x \in [-1, 2) - \{0\}$

Q-8! Solve: $\frac{(x-1)^2(x+1)^3}{x^4(x-2)} \le 0$

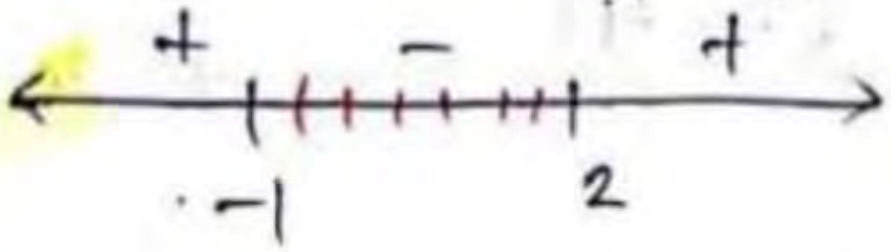
TAH 8

Soln

$$\frac{(x-1)^2(x+1)^3}{x^4\cdot(x-2)} \le 0.$$

$\Rightarrow \frac{(x+1)}{(x-2)} \le 0$ & $\boxed{x=1}$ is also possible

& $\boxed{x \neq 0}$



$\therefore x \in [-1, 2) \cup \{1\} - \{0\}$

$\Rightarrow x \in [-1, 0) \cup (0, 2)$. [Ans]

QUESTION

TAH 09

Find the exhaustive solutions set of

$$\frac{(2x-5)^{100}(x+3)(2x+1)^{101}}{(x^2-4)^{151}(3x-4)^{197}} < 0$$

Ans. $(-\infty, -3) \cup \left(-2, -\frac{1}{2}\right) \cup \left(\frac{4}{3}, 2\right)$

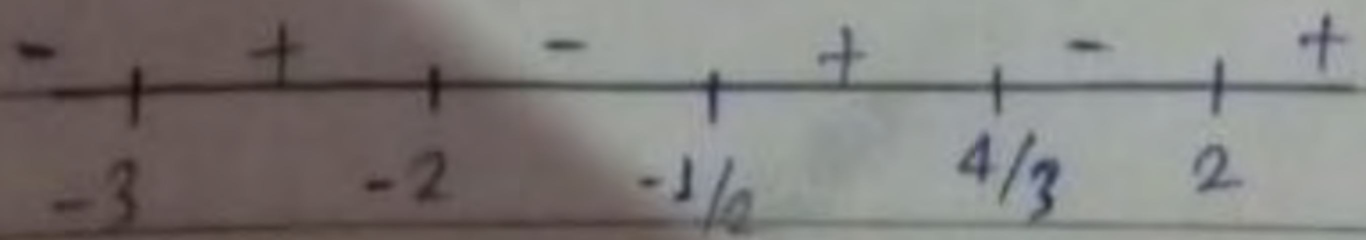
⑨ $$\frac{(2x-5)^{100}(x+3)(2x+1)^{101}}{(x^2-4)^{151}(3x-4)^{197}} < 0$$

$x \neq 5/2 = 2.5$

↳ not in answer

$$\frac{(x+3)(2x+1)^{101}}{(x-2)^{151}(x+2)^{151}(3x-4)^{197}} < 0$$

$x \in (-\infty, -3) \cup (-2, -1/2) \cup (4/3, 2)$



Teacher's Sign

Q-9: Find the exhaustive set of:

$$\frac{(2x-5)^{100}(x+3)(2x+1)^{101}}{(x^2-4)^{151}(3x-4)^{197}} < 0$$

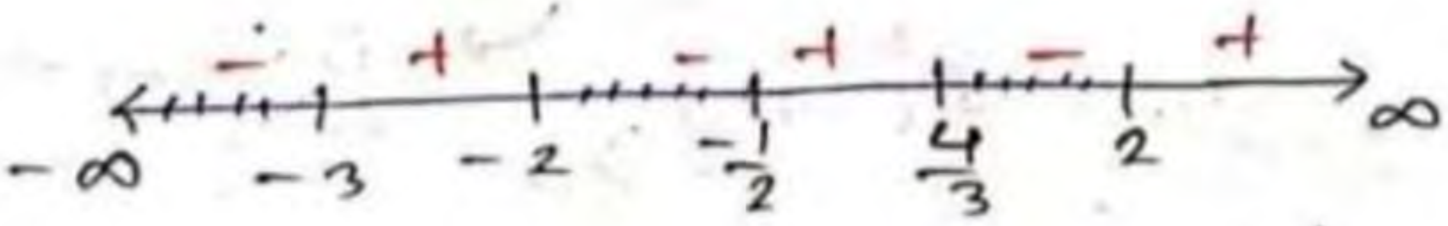
TAH 9
by Reed
from WB

$$\downarrow$$

soln

$$\frac{(x+3)(2x+1)}{(x^2-4)(3x-4)} < 0 \quad \& \quad x \neq \frac{5}{2}$$

$$\Rightarrow \frac{(x+3)(2x+1)}{(x-2)(x+2)(3x-4)} < 0$$



$$\therefore x \in (-\infty,-3) \cup \left(-2,-\tfrac{1}{2}\right) \cup \left(\tfrac{4}{3},2\right) - \left\{\tfrac{5}{2}\right\}$$

$\frac{5}{2} \equiv 2.5$ already not included

$$\Downarrow$$

Ans: $x \in (-\infty,-3) \cup \left(-2,-\tfrac{1}{2}\right) \cup \left(\tfrac{4}{3},2\right)$.

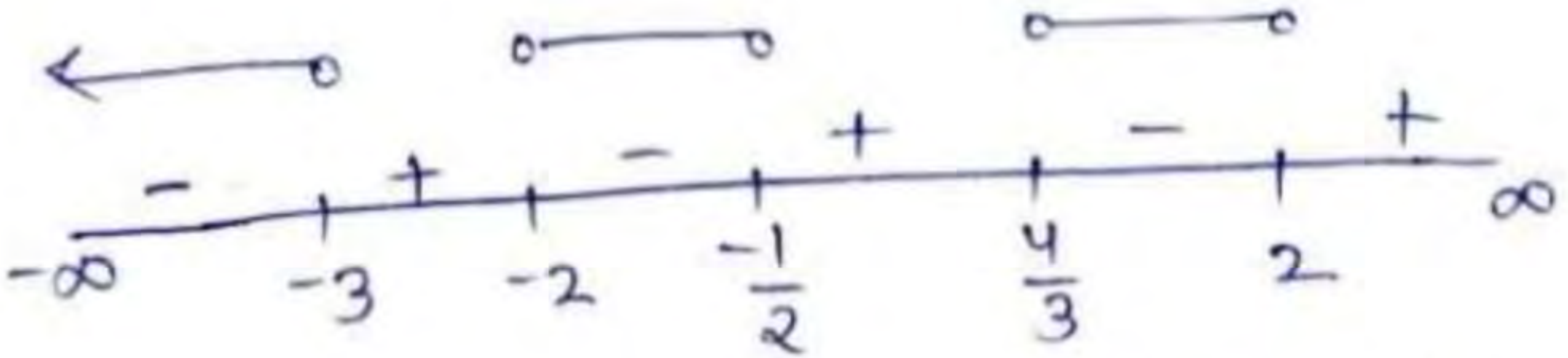
Q Find the exhaustive solutions set of

$$\frac{(2x-5)^{100}(x+3)(2x+1)^{101}}{(x^2-4)^{151}(3x-4)^{197}} < 0$$

Tah-09

$x \neq \frac{5}{2}$

Sol
$$\frac{(x+3)(2x+1)^{101}}{(x-2)^{151}(x+2)^{151}(3x-4)^{197}} < 0$$



$x \in (-\infty, -3) \cup \left(-2, -\frac{1}{2}\right) \cup \left(\frac{4}{3}, 2\right)$ Ans

QUESTION

TAH 10

Solve: $\dfrac{(x^2-4x+5)^2(x-3)^2(x+1)^3}{(x-1)(x-5)^3(x^2-7x+12)} > 0$

Q-10! Solve: $\frac{(x^2-4x+5)^2 (x-3)^2 (x+1)^3}{(x-1)(x-5)^3(x^2-7x+12)} > 0.$

TAH 10

Sol^n ⇒ $D<0$, $a>0$ ⇓ always +ve. ←

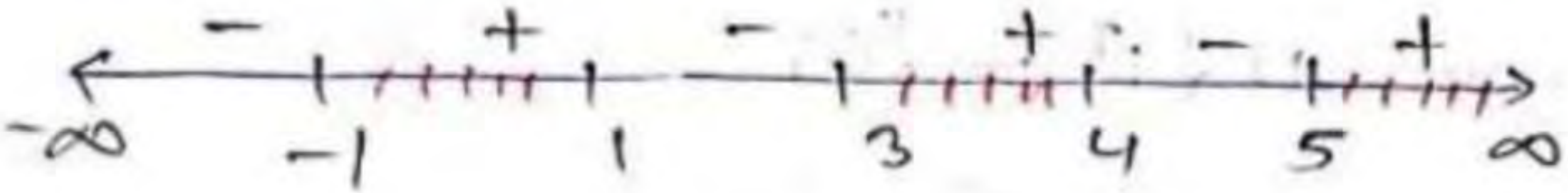
$$\frac{(x^2-4x+5)^2 \cdot (x-3)^2 (x+1)^3}{(x-1)(x-5)^3(x^2-7x+12)} > 0$$

↳ $D>0 \Rightarrow$ factorizable.

$$\Rightarrow \frac{(x+1)^3 (x-3)^{\cancel{2}}}{(x-1)(x-5)^3 \cancel{(x-3)} (x-4)} > 0$$

but $x-3 \neq 0$

$\Rightarrow x \neq 3$



Since $\boxed{0>0}$ is not possible

$\therefore x \in (-1, 1) \cup (3, 4) \cup (5, \infty)$

(Answer.)

Sol

$$\frac{(x^2-4x+5)^2\,(x-3)^2\,(x+1)^3}{(x-1)\,(x-5)^3\,(x^2-7x+12)}>0$$

$(x^2-4x+5)^2$

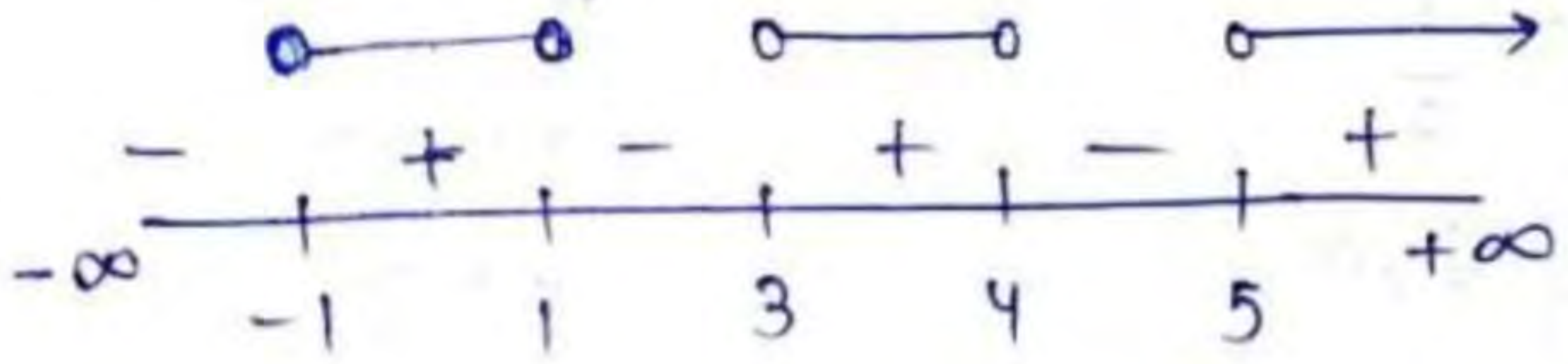
$a>0,\ D<0$

always +ve

$x\neq 3$

Tan-10

$$\frac{(x+1)^3}{(x-1)\,(x-5)^3\,(x-3)(x-4)}>0$$



$x\in(-1,1)\cup(3,4)\cup(5,\infty)$ **Ans**

QUESTION

TAH 11

Find the exhaustive solutions set of $\frac{(x-4)(2x-5)^{27}\left(x^2-9\right)^{10}(x+4)^{93}}{(x^2-25)(x+3)^{91}(x^2+10)^5} > 0$.

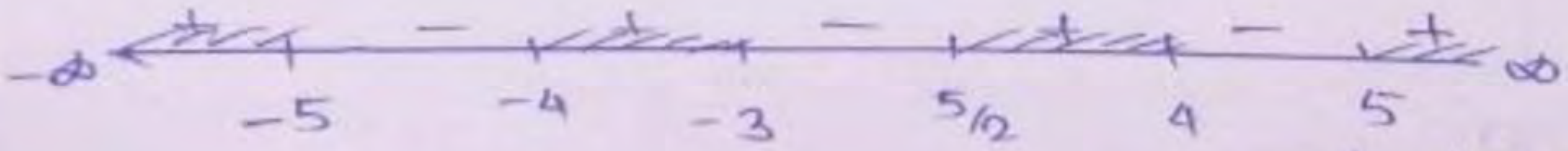
Ans. $(-\infty, -5) \cup (-4, -3) \cup \left(\frac{5}{2}, 3\right) \cup (\mathbf{3}, \mathbf{4}) \cup (\mathbf{5}, \infty)$

TAH-11

Q. Find exhaustive soln. set of $\frac{(x-4)(2x-5)^{27}(x^2-9)^{10}(x+4)^{93}}{(x^2-25)(x+3)^{91}(x^2+10)^5} > 0$

$$\Rightarrow \frac{(x-4)(2x-5)^{27}(x+3)^{10}(x-3)^{10}(x+4)^{93}}{(x+5)(x-5)(x+3)^{91}\underbrace{(x^2+10)^5}_{\substack{a>0,\,D<0 \\ +\text{ve always}}}} > 0$$

$$\Rightarrow \frac{(x-4)(2x-5)^{27}(x+4)^{93}}{(x+5)(x-5)(x+3)^{81}} > 0$$, $x \neq -3$, $x = 3$ is not possible as, it not satisfy inequality.



$$\therefore\ x \in (-\infty,-5) \cup (-4,\overset{\text{already excluded}}{-3}) \cup (5/2, 4) \cup (5,\infty) - \{3\}$$

$$\text{or, } (-\infty,-5) \cup (-4,-3) \cup (5/2, 3) \cup (3,4) \cup (5,\infty)$$

Ans

• Q-11:- Find the exhaustive solⁿ set of:

$$\frac{(x-4)(2x-5)^{27}(x^2-9)^{10}(x+4)^{93}}{(x^2-25)(x+3)^{91}(x^2+10)^5} > 0.$$

TAH 11
by Reed
from WB

Solⁿ:

$$\frac{(x-4)(2x-5)^{27}(x^2-9)^{10}(x+4)^{93}}{(x^2-25)(x+3)^{91}(x^2+10)^5} > 0$$

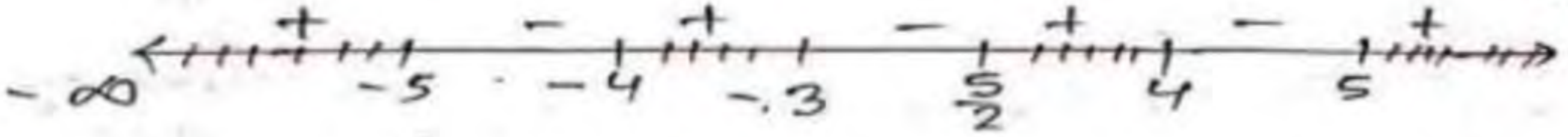
$$\Rightarrow \frac{(x-4)(2x-5)(x+3)^{10}(x-3)^{10}(x+4)}{(x+5)(x-5)(x+3)^{91}(x^2+10)^5} > 0$$

$(x^2+10)^5 \hookrightarrow D<0,\ a>0 \Rightarrow$ always +ve.

$$\Rightarrow \frac{(x-4)(2x-5)\overset{1}{\cancel{(x-3)^{10}}}(x+4)}{(x+5)(x-5)(x+3)^{81}\cdot 1} > 0$$

& $x \neq -3$.

$x \neq 3$.



$\therefore x \in (-\infty,-5) \cup (-4,-3) \cup (\frac{5}{2},4) \cup (5,\infty) - \{3\}$.

↳ already excluded.

$\Rightarrow x \in (-\infty,-5) \cup (-4,-3) \cup (\frac{5}{2},3) \cup (3,4) \cup (5,\infty)$

Answer.

TAH - 11

Find the exhaustive solⁿ set of

$$\frac{(x-4)(2x-5)^{27}(x^2-9)^{10}(x+4)^{93}}{(x^2-25)(x+3)^{91}(x^2+10)^5} > 0$$

solⁿ

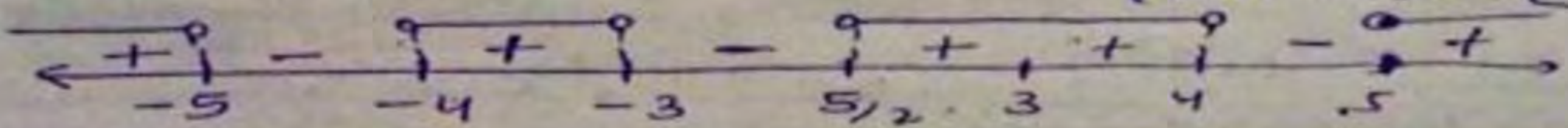
$$\frac{(x-4)(2x-5)^{27}(x-3)^{10}\cancel{(x+3)^{10}}(x+4)^{93}}{(x+5)(x-5)(x+3)^{\cancel{91}\,81}\underbrace{(x^2+10)^5}_{+}} > 0$$

$$\frac{(x-4)(2x-5)^{27}(x-3)^{10}(x+4)^{93}}{(x+5)(x-5)(x+3)^{81}\cancel{(x^2+10)^5}} > 0$$

$\boxed{x = 3 \text{ is eleminated}}$
(+ve region)

$D = 0 - 40$
$D = -40$
$D < 0,\ a > 0$
$\therefore x^2+10 > 0 \ \forall$
$x \in R$

$\therefore$



(for 3 as a critical point sign will remain same and it is excluded as for $x = 3$, $0 > 0$ is false)

$$\boxed{\begin{array}{l} x \in (-\infty, 5) \cup (-4,-3) \cup (5/2, 4) \cup (5,\infty) - \{3\} \\ \text{OR} \\ x \in (-\infty, 5) \cup (-4,-3) \cup (\tfrac{5}{2}, 3) \cup (3,4) \cup (5,\infty) \end{array}}$$

ans

QUESTION

TAH 12

Solve the system of in equations $2x - 1 > x + \frac{7-x}{3} > 2, x \in R$

TAH-12

Q. Solve the system of inequations $2x-1 > x+\frac{7-x}{3} > 2$, $x \in R$.

$\Rightarrow \quad 2x-1 > x+\frac{7-x}{3} > 2$

$\Rightarrow \quad 6x-3 > 3x+7-x > 6$

$\Rightarrow \quad 6x-3 > 2x+7 > 6$

$\Rightarrow \quad 6x-3-7 > 2x > 6-7$

$\Rightarrow \quad 6x-10 > 2x > -1$

$\Rightarrow \quad 3x-5 > x > -1/2$

$\Rightarrow \quad 3x-5 > x$ or, $x > -\frac{1}{2}$

$\Rightarrow \quad 2x > 5 \quad \cap \quad x > -1/2$

$\Rightarrow \quad \underbrace{x > 5/2 \quad \cap \quad x > -1/2}$

$\therefore \quad x > 5/2$ or, $x \in \left(\frac{5}{2}, \infty\right)$ Ans//

• Q-12!
TAH-12

Solve the system of inequn.

$$2x-1 > x+\frac{7-x}{3} > 2;\quad x\in R.$$

Sol^n ⇒

$$2x-1 > x+\frac{7-x}{3} > 2$$

$$\Rightarrow\quad 6x-3 > \frac{(2x+7)\cancel{3}}{\cancel{3}} > 6.$$

$$\Rightarrow\quad 6x-3 > 2x+7 > 6$$

$$\Rightarrow\quad 6x-3-7 > 2x > 6-7$$

$$\Rightarrow\quad 6x-10 > 2x > -1$$

$$\Rightarrow\quad 3x-5 > x > -\frac{1}{2}$$

$$\Rightarrow\quad -\frac{1}{2} < x < 3x-5.$$

or, $\boxed{x > -\frac{1}{2}}$

$3x-5 > x$

or, $2x > 5$

or, $\boxed{x > \frac{5}{2}}$

"∩" ⇒ $\boxed{x > \frac{5}{2}}$ $\therefore x \in \left(\frac{5}{2}, \infty\right)$ Ans.

TAH 12
by Reed
from WB

QUESTION

TAH 13

Solve following double inequalities :

(i) $-3 < \frac{2x-7}{5} \leq 8$

(ii) $x^2 + 2 \leq 3x < 2x^2 - 5$

(iii) $-2 < \frac{x-5}{2x+1} < 5$

TAH-13

TAH-13(i), (ii) BY
NEELAKSH THAKUR
SUPAUL

Q. Solve the following inequalities:-

(i). $-3 < \frac{2x-7}{5} \leq 8$ (ii). $x^2+2 \leqslant 3x \leqslant 2x^2-5$ (iii). $-2 < \frac{x-5}{2x+1} < 5$

⇒ (i). $-3 < \frac{2x-7}{5} \leq 8$

$\Rightarrow -15 < 2x-7 \leqslant 8\times 5$

$\Rightarrow -15+7 < 2x \leqslant 40+7$

$\Rightarrow -8/2 < x \leqslant 47/2$

$\therefore -4 < x \leqslant 47/2$

or, $x \in (-4, 47/2]$

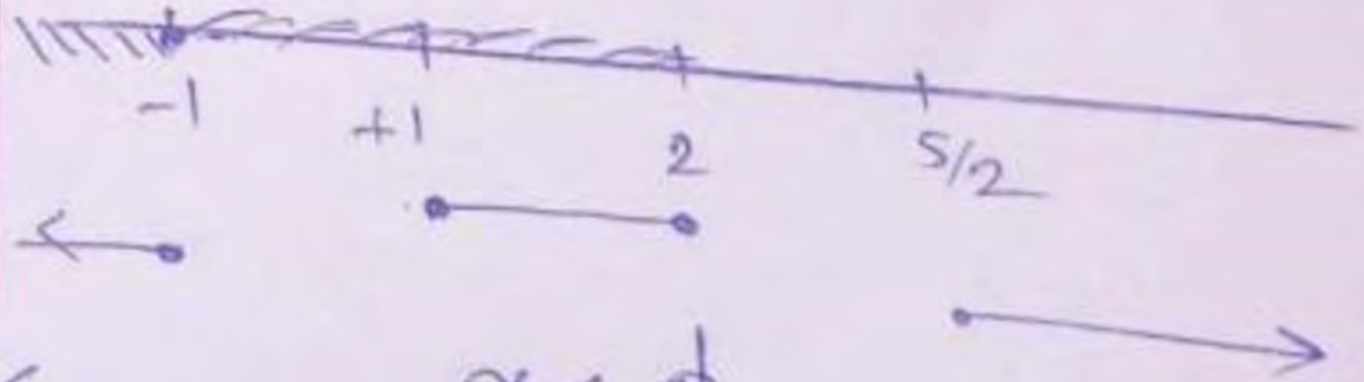
Ans //

(ii). $x^2+2 \leqslant 3x \leqslant 2x^2-5$

$x^2+2-3x \leqslant 0 \cap 2x^2-5-3x \geqslant 0$

$(x-1)(x-2) \leqslant 0 \cap (x+1)(2x-5) \geqslant 0$

$x \in [1, 2] \cap x \in (-\infty, -1] \cup [5/2, \infty)$



$x \in \phi$

Ans

Q-13! Solve:

TAH 13
BY REED
FROM WB

(i) $-3 < \frac{2x-7}{5} \le 8$

$\Rightarrow -15 < 2x-7 \le 40$

$\Rightarrow -8 \le 2x \le 47$

$\Rightarrow -4 \le x \le \frac{47}{2}$ $\therefore x \in [-4, \frac{47}{2}]$

(ii) $x^2+2 \le 3x < 2x^2-5$

$x^2+2-3x \le 0$	$2x^2-5-3x > 0$
$\Rightarrow (x-2)(x-1) \le 0$	$\Rightarrow 2x^2-3x-5 > 0$
$\Downarrow$	$\Rightarrow (2x-5)(x+1) > 0$
$x \in [1, 2]$	$\Downarrow$
	$x \in (-\infty, -1) \cup (\frac{5}{2}, \infty)$

$\cap$

$\therefore \boxed{x \in \phi}$

(iii) ⇒ $-2 < \frac{x-5}{2x+1} < 5$

TAH-13 (iii) BY
NEELAKSH THAKUR
SUPAUL

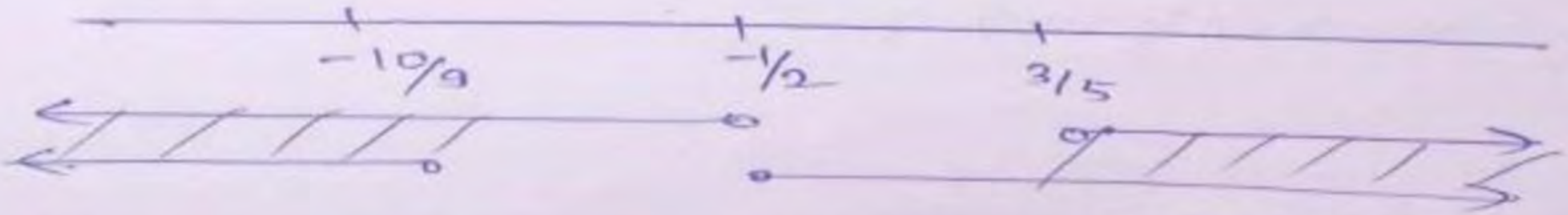
⇒ $-2 < \frac{x-5}{2x+1} \cap \frac{x-5}{2x+1} < 5$

⇒ $\frac{x-5+4x+2}{2x+1} > 0 \cap \frac{x-5-10x-5}{2x+1} < 0$

⇒ $\frac{5x-3}{2x+1} > 0 \cap \frac{-9x-10}{2x+1} < 0$

⇒ $\frac{5x-3}{2x+1} > 0 \cap \frac{9x+10}{2x+1} > 0$

$x \in (-\infty, -1/2) \cup (3/5, \infty) \cap x \in (-\infty, -\frac{10}{9}) \cup (-1/2, \infty)$



∴ $x \in (-\infty, -10/9) \cup (3/5, \infty)$

Ans

Q11 $-2 < \dfrac{x-5}{2x+1} < 5$

&

$\Rightarrow \dfrac{x-5}{2x+1} > -2$

$\Rightarrow \dfrac{x-5+4x+2}{2x+1} > 0$

$\Rightarrow \dfrac{5x-3}{2x+1} > 0$

$\Downarrow$

$x \in \left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{3}{5}, \infty\right)$

$\dfrac{x-5}{2x+1} < 5$

$\Rightarrow \dfrac{x-5-10x-5}{2x+1} < 0$

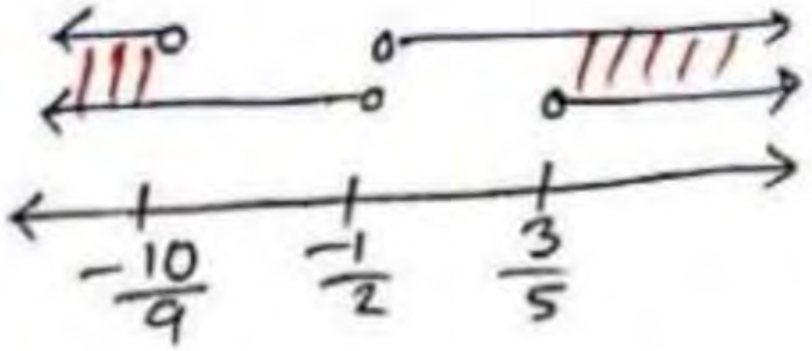
$\Rightarrow \dfrac{9x+10}{2x+1} > 0$

$\Downarrow$

$x \in \left(-\infty, -\frac{10}{9}\right) \cup \left(-\frac{1}{2}, \infty\right)$

$\cap$

$\therefore x \in \left(-\infty, -\frac{10}{9}\right) \cup \left(\frac{3}{5}, \infty\right)$



Answer

THANK YOU